UNIVERSITY OF LONDON | INTERNATIONAL PROGRAMMES

Course information 2015–16
EC2065 Macroeconomics

This course introduces students to the most influential and compelling theories designed by macroeconomists to explain issues related to the determination of output, unemployment and inflation. Students will acquire a logical and consistent framework for understanding the main macroeconomic facts and events, and develop the ability to employ the correct macroeconomic tool(s) to explain specific macroeconomic issues and justify policy proposals.

Prerequisite

If taken as part of a BSc degree, courses which must be passed before this course may be attempted:

EC1002 Introduction to economics
And either
MT105a Mathematics 1
or
MT1174 Calculus

Aims and objectives

The aims of the course are to:

- show how our understanding of how economic systems operate has evolved substantially
- explain why the growth rate of aggregate output varies from year to year
- explain what determines unemployment and inflation in the short run and in the long run
- discuss how macroeconomic policy might influence business cycles or long run growth.

Assessment

This course is assessed by a three hour unseen written examination.

Learning outcomes

At the end of this course and having completed the essential reading and activities students should be able to:

- ☑ Define and analyse the determinants of business cycles, long run economic growth, unemployment, inflation.
- ☑ Use and apply a wide range of economic models to analyse contemporary and historical macroeconomic events, and formulate and propose appropriate macroeconomic policies.

Essential reading

For full details please refer to the reading list.

Blanchard, O. *Macroeconomics*. (Prentice Hall)
Dornbusch, R., S. Fischer and R. Startz *Macroeconomics*. (McGraw-Hill)
Mankiw, N.G. *Macroeconomics*. (Worth)

Students should consult the *Programme Regulations for degrees and diplomas in Economics, Management, Finance and the Social Sciences* that are reviewed annually. Notice is also given in the *Regulations* of any courses which are being phased out and students are advised to check course availability.

Syllabus

This is a description of the material to be examined, as published in the *Programme handbook*. On registration, students will receive a detailed subject guide which provides a framework for covering the topics in the syllabus and directions to the essential reading.

This course covers the main principles involved in the determination of real income, employment and unemployment, the price level and inflation in an open mixed economy, and the conduct of macroeconomic policy.

The main topics are:

- Aggregate demand in a closed economy: the determinants of consumption, investment, demand for and supply of money; wealth effects; the IS-LM model and policy prescriptions.
- Aggregate demand in an open economy: exchange rate regimes, international trade and capital flows, and external balance; the IS-LM-BP model and policy prescriptions.
- Aggregate demand, aggregate supply and the price level: the aggregate demand curve; short and long run aggregate supply curves; the aggregate demand-aggregate supply model and its applications to the determination of the price level and real income, and demand management policy; the neo-classical (Solow) growth model.
- Inflation and unemployment; models of inflation; costs of inflation; counter-inflationary policy; full employment and the natural rate of unemployment; types and causes of unemployment, and policies to reduce them.

Examiners' commentaries 2015

EC2065 Macroeconomics

Important note

This commentary reflects the examination and assessment arrangements for this course in the academic year 2014–15. The format and structure of the examination may change in future years, and any such changes will be publicised on the virtual learning environment (VLE).

Information about the subject guide and the Essential reading references

Unless otherwise stated, all cross-references will be to the latest version of the subject guide (2014). You should always attempt to use the most recent edition of any Essential reading textbook, even if the commentary and/or online reading list and/or subject guide refer to an earlier edition. If different editions of Essential reading are listed, please check the VLE for reading supplements – if none are available, please use the contents list and index of the new edition to find the relevant section.

Comments on specific questions – Zone A

Candidates should answer **ELEVEN** of the following **FOURTEEN** questions: all **EIGHT** from Section A (5 marks each) and **THREE** from Section B (20 marks each). **Candidates are strongly advised to divide their time accordingly.**

If more questions are answered than requested, only the first answers attempted will be counted. PLEASE

Section A

Answer **all eight** questions in this section (5 marks each).

Briefly explain whether each of the following statements is true or false.

Question 1

'If the demand for money does not depend on income then the LM curve is vertical.'

Reading for this question

Subject guide, Chapter 2.

Approaching the question

The statement is **FALSE**.

The LM curve represents money-market equilibrium where the (real) supply of money is equal to

the (real) demand for money:

$$\frac{M^s}{P} = L(Y, i) = L(i).$$

The money supply M^s is fixed and the price level P is given. Demand for real money balances $L(Y, i)$ does not depend on Y, so money demand is written as $L(i)$. There is a nominal interest rate i consistent with money-market equilibrium. This interest rate does not depend on real income Y, which does not appear in the equation for money-market equilibrium. Thus, the LM curve comprises all points with the equilibrium nominal interest rate and any level of real income, that is, a horizontal line at the equilibrium interest rate.

Question 2

'The life-cycle theory of consumption is inconsistent with the fact that the average propensity to consume is lower for households with higher incomes.'

Reading for this question

Subject guide, Chapter 9.

Approaching the question

The statement is **FALSE**.

The life-cycle theory of consumption is based on the idea that households would like a relatively smooth path of consumption even though their (labour) incomes may change at different points in their life (low income when young, high income when middle-aged, and no income in retirement). Consumption smoothing means using borrowing and saving to achieve a consumption path that is more stable than income. Thus, according to the theory, middle-aged households with high incomes may have similar levels of consumption to older retired households with little income. The average propensity to consume (APC $= C/Y$) of the former is lower than the latter, consistent with the data.

Question 3

'A country that runs a current-account deficit must have a capital-account surplus.'

Reading for this question

Subject guide, Chapter 7.

Approaching the question

The statement is **TRUE**.

A country that runs a current-account deficit imports more than it exports. This means it must pay for some of its imports not from its export earnings, but by selling assets (whether held domestically or overseas) to foreigners, or strictly speaking, selling more assets to foreigners than are bought from foreigners. This is the definition of a capital-account surplus. Thus, given a zero balance of payments, which comprises the current and capital accounts, a deficit on one account must be matched (in equilibrium) by a surplus on the other account.

Question 4

'Long-run economic growth in the Solow model is exogenous.'

Reading for this question

Subject guide, Chapter 6.

Approaching the question

The statement is **TRUE.**

Given diminishing marginal returns to capital, growth in real GDP per person cannot be sustained indefinitely through capital accumulation alone. This leaves changes in productivity to drive long-run growth. However, productivity growth is introduced into the Solow model by assumption: it is not explained by the model and is thus an exogenous variable. This means that long-run growth is exogenous in the Solow model.

Question 5

'Minimum wages can be a cause of classical unemployment.'

Reading for this question

Subject guide, Chapter 3.

Approaching the question

The statement is **TRUE**.

Classical unemployment is caused by real wages being unable to fall to a level that bring the demand for labour into line with the supply of labour. Minimum wages prevent firms and workers agreeing to employment contracts below the minimum wage, so the market wage might be unable to fall by enough to clear the labour market.

Question 6

'National saving in a closed economy is equal to the sum of investment and private saving.'

Reading for this question

Subject guide, Chapter 2.

Approaching the question

The statement is **FALSE**.

National saving S_N is defined as the sum of private saving S_P and public saving S_G. Private saving is disposable income $Y - T$ minus consumption C, so $S_P = Y - T - C$, and public saving is the difference between tax receipts T and public expenditure G, so $S_G = T - G$. Using these definitions, it follows that national saving is:

$$S_N = (Y - T - C) + (T - G) = Y - C - G$$

and since $Y = C + I + G$ is goods-market equilibrium in a closed economy:

$$S_N = (C + I + G) - (C + G) = I.$$

This shows that in equilibrium, national saving is simply equal to investment in a closed economy.

Question 7

'Interest rates (yields) are high when bond prices are low.'

Reading for this question

Subject guide, Chapter 10.

Approaching the question

The statement is **TRUE**.

The yield i on a simple discount bond is the (annual) percentage return if the bond is held to maturity. For a simple discount bond with face value 1, the percentage gain if the bond is bought at price Q and 1 is paid at maturity is:

$$i = \frac{1-Q}{Q} \quad \Rightarrow \quad Q = \frac{1}{1+i}.$$

This shows that bond prices and interest rates are inversely related, so low bond prices correspond to high interest rates (yields).

Question 8

'An increase in the natural rate of unemployment shifts the Phillips curve to the left.'

Reading for this question

Subject guide, Chapter 4.

Approaching the question

The statement is **FALSE**.

The expectations-augmented Phillips curve passes through the point where unemployment is equal to the natural rate of unemployment and actual inflation is equal to expectations of inflation (plus any cost-push shocks). With no change in inflation expectations or cost-push shocks, an increase in the natural rate of unemployment means that the same inflation rate now results in higher unemployment, or equivalently, the same unemployment rate now results in higher inflation. This means that the Phillips curve shifts to the right.

Section B

Answer **three** questions from this section (20 marks each).

Question 9

Consider a small open economy with fixed prices and wages.

Goods-market equilibrium is where output Y is equal to the sum of consumption C, investment I, government spending G, and net exports NX. The consumption and investment functions are:

$$C = C_0 + c(Y - T), \quad I = I_0 - bi$$

where i is the domestic interest rate. Government spending and taxes are exogenously fixed at $G = G_0$ and $T = T_0$. Net exports are given by:

$$NX = NX_0 - mY - ae$$

where e is the nominal exchange rate (defined as the foreign-currency price of domestic currency).

Money-market equilibrium is where the demand for real money balances $L(Y, i)$ is equal to the real money supply M^s/P:

$$\frac{M^s}{P} = L(Y, i)$$

where $L(Y, i)$ is an increasing function of Y and a decreasing function of i.

Capital mobility is perfect, so balance-of-payments equilibrium requires that the domestic interest rate i is equal to the foreign interest rate i^*. Throughout the question, assume the exchange rate is flexible.

(a) [7 marks] Use the IS–LM–BP diagram to find the new point of goods, money, and balance-of-payments equilibrium after the following changes, and explain the effects of each on output, the exchange rate, and the current account (net exports):

i. An expansion of the domestic money supply M^s.

ii. Looser foreign monetary policy, which reduces the foreign interest rate i^*.

(b) [6 marks] Assume that the demand for money $L(Y, i)$ becomes perfectly interest-elastic when the interest rate i falls to zero. Explain why this means that the LM curve becomes horizontal where the interest rate is zero. Find the effects on the LM curve of an increase in the money supply, taking account of the behaviour of money demand at zero interest rates.

(c) [7 marks] Suppose that the foreign interest rate falls to zero ($i^* = 0$). Starting from the new equilibrium after adjusting to $i^* = 0$, find the effects of an expansion of the domestic money supply on output, the exchange rate, and the current account (net exports). Explain the intuition behind your findings.

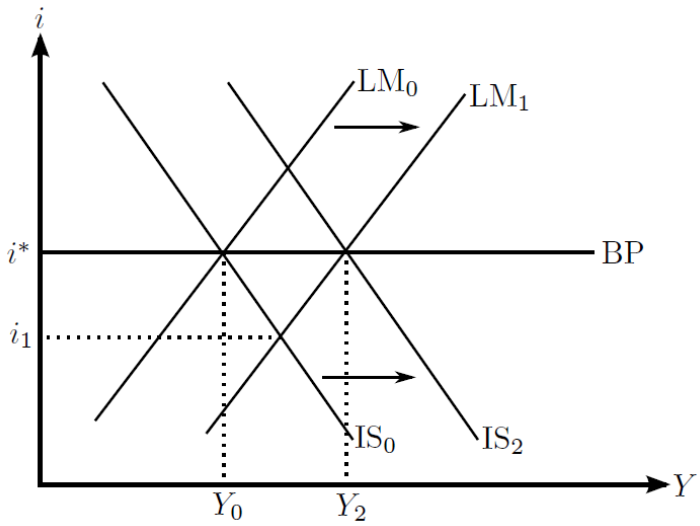
Reading for this question

Subject guide, Chapters 4, 7 and 8.

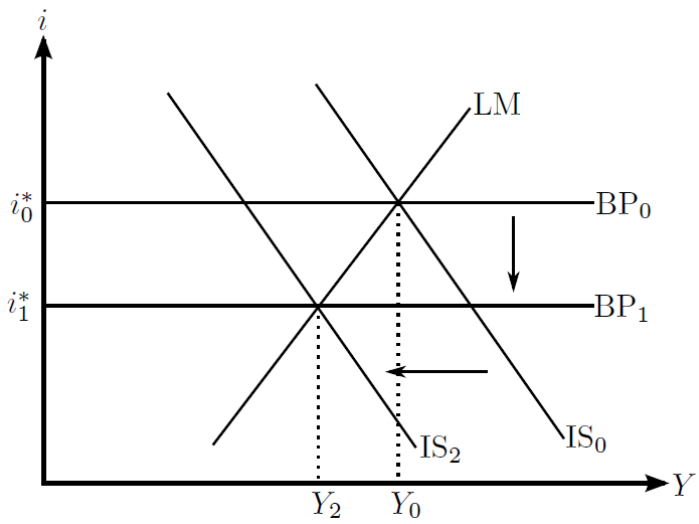
Approaching the question

(a) The equilibrium of the economy is found using the IS–LM–BP diagram. The BP curve is horizontal at the foreign interest rate i^* because capital mobility is perfect.

An increase in the money supply shifts the LM curve to the right. Without a change in the exchange rate, goods and money-market equilibrium (the intersection of the IS and LM curves) now requires an interest rate i_1 below the foreign interest rate i^*. This would result in capital outflows, which put downward pressure on the exchange rate. As the exchange rate depreciates, since prices are sticky, this leads to an improvement in competitiveness and thus net exports (given the Marshall–Lerner condition), shifting the IS curve to the right. The rightward shift of IS increases the interest rate consistent with goods and money-market equilibrium until it reaches the foreign interest rate and there is no incentive for further capital flows. Output increases, the exchange rate depreciates, and the current account increases.



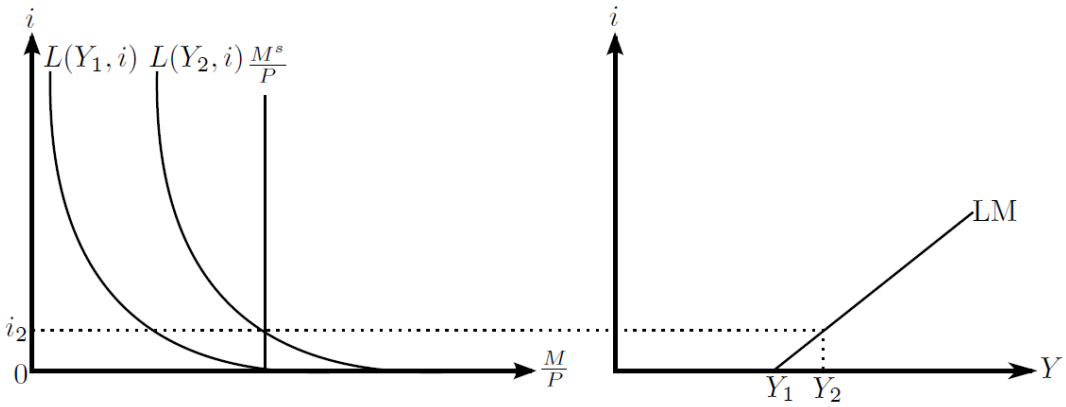
Now consider what happens if the monetary expansion occurs in the rest of the world, leading to a lower foreign interest rate i^*. This shifts down the BP curve because investors are now willing to keep their funds invested in domestic assets at a lower interest rate. If there was no change in the exchange rate, goods and money-market equilibrium would continue to occur at the former interest rate i_0^*, which is now above the new foreign interest rate i_1^*. This would lead to capital inflows, which put upward pressure on the exchange rate. As the exchange rate rises and competitiveness deteriorates, net exports fall, shifting the IS curve to the left. Equilibrium in goods, money, and foreign-exchange markets is restored when the exchange rate has appreciated by enough to shift the IS curve to a point where it intersects the LM curve and the new BP curve. Output falls, the exchange rate appreciates, and the current account decreases.



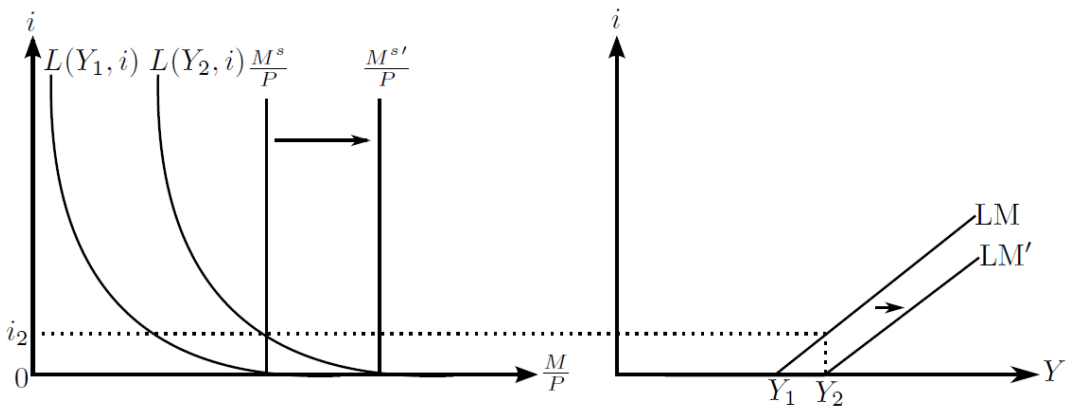
(b) The LM curve is derived by finding the nominal interest rate that achieves money-market equilibrium for different levels of output (which shift money demand). Money demand is a decreasing function of the nominal interest rate, and it becomes perfectly elastic (the demand curve becomes horizontal) when the interest rate falls to zero.

If there is an increase in income from Y_1 to Y_2 then the money demand curve shifts to the right. When the nominal interest rate is positive, money-market equilibrium requires a rise in the nominal interest rate (the supply of money is constant), implying that the LM curve is upward sloping. However, if money demand is too low at the prevailing level of income, equilibrium is reached where the money demand curve becomes perfectly interest-elastic at zero interest rates. Any further reduction in money demand owing to lower income has no

effect on the equilibrium interest rate. This means that the LM curve has a horizontal section at a nominal interest rate of zero.

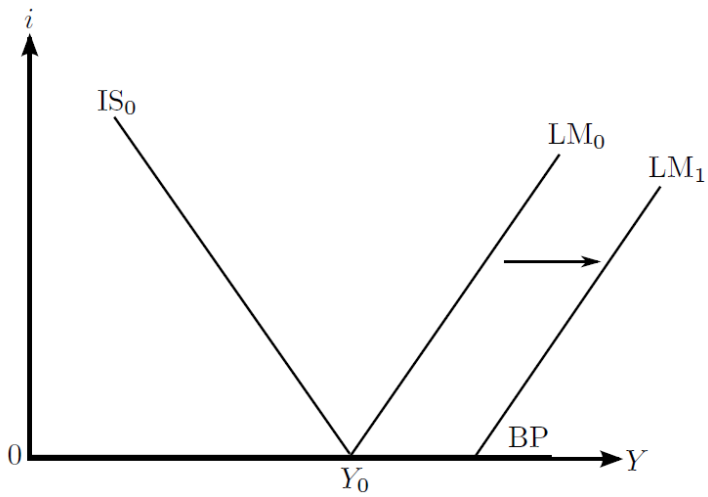


An increase in the money supply lowers the equilibrium nominal interest rate when this is initially positive. This means that the monetary expansion shifts the LM curve to the right. However, at income levels where the LM curve is on the horizontal axis, the additional money is simply absorbed by money demand with no change in the nominal interest rate. This means that the horizontal section of the LM curve expands as it shifts to the right. It does not shift down below the horizontal axis.



(c) Using the analysis from part (a), as the foreign interest rate falls, the BP curve would shift down and the IS curve would shift to the left as the exchange rate appreciates. Once the foreign interest rate has reached zero, the domestic economy is at a point where IS and LM intersect at an interest rate of zero, which is where the BP curve is now positioned.

An increase in the domestic money supply shifts the LM curve to the right. Using the analysis in part (b), the LM curve is horizontal at zero interest rates, so the horizontal section expands as the LM curve shifts to the right (this section does not shift down, unlike the section of the LM where interest rates are positive). As a result, equilibrium in domestic goods and money markets still occurs at an interest rate of zero. With no change in the interest rate, there is no incentive for capital to flow out in search of higher returns in other countries, unlike what occurred in part (a). As a result, there is no pressure on the exchange rate to depreciate, and thus no shift of the IS curve. The equilibrium of the economy remains at an interest rate of zero and output of Y_0 (the only difference is that IS intersects LM on its horizontal section to the left of its kink). The monetary expansion has no effect on the economy even though the exchange rate is flexible.



While this answer is correct, given the transmission mechanism of monetary policy embedded in the IS–LM–BP model, it seems implausible that a central bank that engages in potentially unlimited selling of its currency (expansion of the money supply, acquiring foreign exchange reserves) would be unable to affect its exchange rate. This may be so, but logically, it would require that the transmission of monetary policy works through different channels (expectations, risk premia, etc.) from those in the IS–LM–BP model.

Question 10

Consider the standard IS–LM model in a closed economy. Consumption depends positively on disposable income and investment depends negatively on the real interest rate. The demand for real money balances depends positively on income and negatively on the nominal interest rate.

(a) [6 marks]

i. Explain intuitively why nominal money demand is assumed to be proportional to the price level.

ii. Find the effects on output and interest rates of an increase in the price level when the nominal money supply remains constant (assume that expected inflation remains zero, so the real interest rate is equal to the nominal interest rate).

iii. Explain what is meant by the term 'real balance effect', and show how the inclusion of real balance effects in the IS–LM model would affect your answer.

(b) [7 marks]

i. Carefully explain why households and firms are willing to hold money that pays no interest when the nominal interest rate on bonds is positive. Use your answer to explain why the demand for money balances depends negatively on the nominal interest rate, and why the demand for money becomes perfectly interest-elastic at zero nominal interest rates.

ii. How would the demand curve for money be affected if there were a cost of storing money, but no cost of storing bonds? Explain.

(c) [7 marks] Suppose that prices are expected to fall in the future, meaning that expected inflation becomes negative.

i. Write down and explain the Fisher equation that links nominal and real interest rates to expected inflation, and assuming there is no change in monetary policy, use the IS–LM model to find the effects of the expected deflation on output and the nominal and real interest rates.

ii. How would your answers be different if monetary policy reacts to changes in expected inflation and satisfies the 'Taylor principle' where nominal interest rates move by more than changes in (expected) inflation?

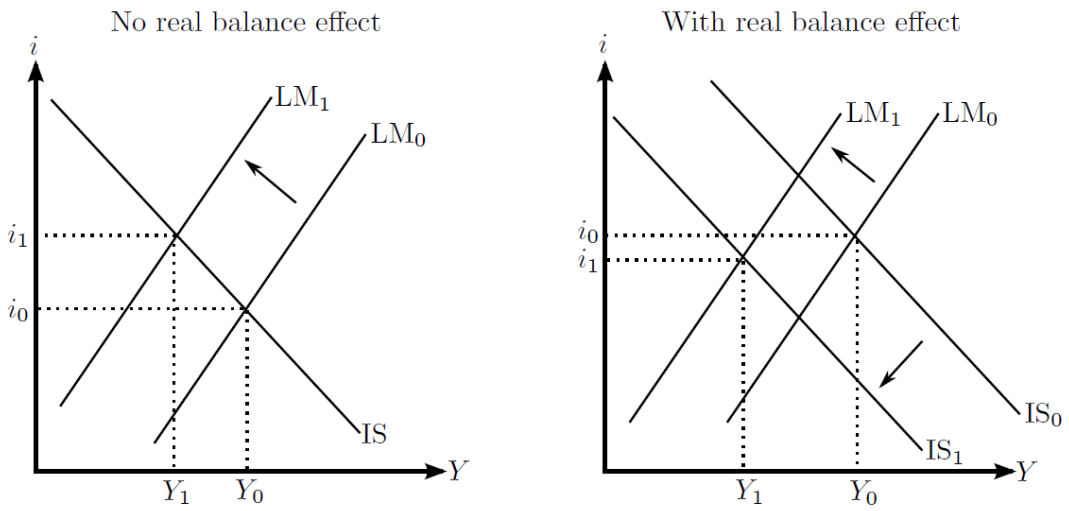
Reading for this question

Subject guide, Chapters 10 and 12.

Approaching the question

(a) Nominal money demand depends positively on the price level because money is held for its usefulness in performing transactions (it is dominated in return by other assets such as bonds when interest rates are positive). The need to hold money increases when the price level is higher if the same volume of real transactions is to be performed.

An increase in the price level decreases the real money supply. Given real income Y, the nominal interest rate i must rise to decrease real money demand in line with supply. This shifts the LM curve to the left. With no effect on the IS curve (in a closed economy with no real balance effects), this lowers output and raises the interest rate.



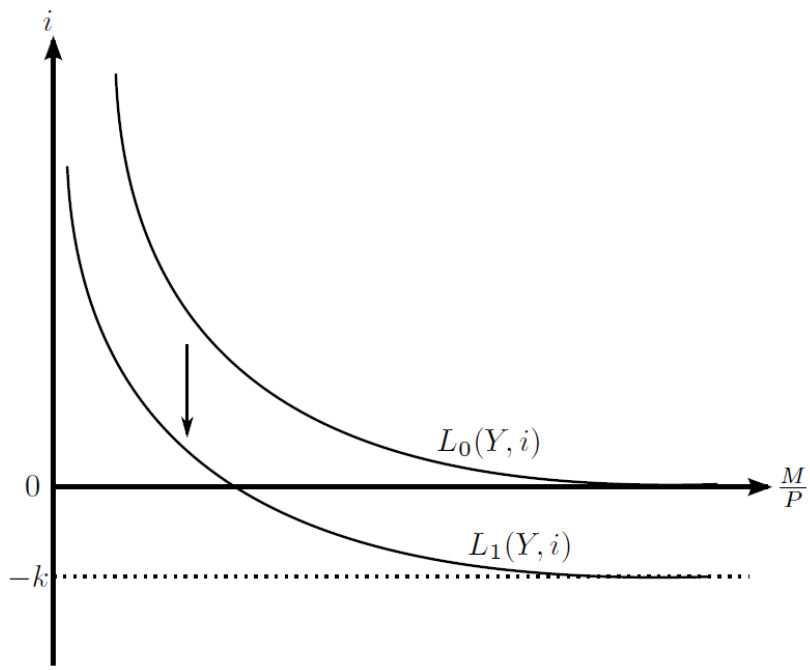
A real balance effect is present when consumption demand depends on holdings of real money balances. The argument is that these are a component of households' net wealth, so an increase in their real value increases consumption through a wealth effect. As a result, when the price level rises, real money balances fall and the IS curve shifts to the left owing to a decrease in consumption demand. The effect of the price level on the LM curve is the same as before. Overall, output still decreases (by more in this case), but now the effect on the interest rate is uncertain.

(b) The nominal interest rate is the difference between the return on bonds and the return on money that pays no interest. It is thus the opportunity cost of holding money relative to interest-bearing assets. Households are willing to hold some wealth in the form of money as long as money provides a service that compensates for its low return relative to other assets. This is its usefulness in performing transactions (money is readily acceptable for payments, unlike other assets for which there are transaction costs). The marginal value of this service depends on how much money is held: with large holdings of money, an extra unit contributes little in facilitating transactions, and thus it will be willingly held only if the opportunity cost (the nominal interest rate) is small. This explains why money demand is a decreasing function of the nominal interest rate.

While the marginal value of money in performing transactions diminishes as more money is held, the services it provides never have a negative value. Thus, once the nominal interest

rate has reached zero, there is no opportunity cost of holding money, so effectively unlimited amounts of wealth can be rationally held as money (other assets do not offer a better return). This means the money demand curve becomes perfectly interest elastic at zero nominal interest rates.

If there were a cost of storing money (but no cost of storing bonds) then this storage cost (denoted by k as a fraction of the value of the money stored) must be added to the nominal interest rate i to yield the total cost of holding wealth as money instead of bonds. Money is only held to the extent that its marginal value in performing transactions is at least $i + k$. In this case, bonds can be rationally held even when $i < 0$ as long as $i + k \geq 0$. It follows that nominal interest rates can now fall below zero, but cannot fall below $-k$. The money demand curve shifts down and now has a section below the horizontal axis.

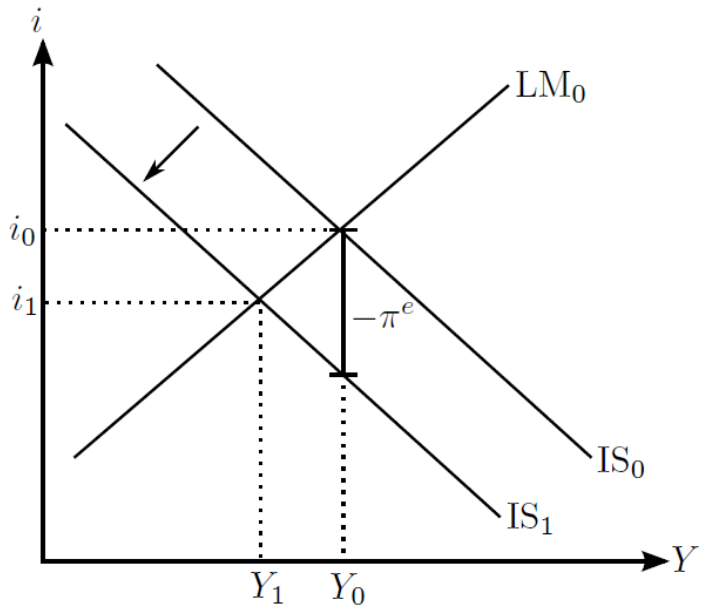


(c) The Fisher equation is:

$$i = r + \pi^e$$

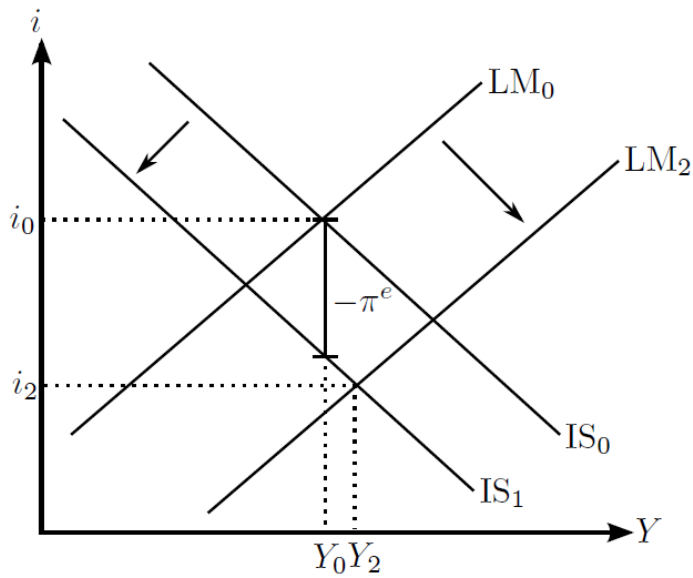
where i is the nominal interest rate, r is the real interest rate, and π^e is expected inflation. The Fisher equation follows by calculating the real return implied by a nominal bond paying interest rate i. The expected real return, which is r, is obtained by subtracting any expected inflation π^e.

When prices are expected to fall in the future, expected inflation is negative ($\pi^e < 0$). This means that nominal and real interest rates are no longer equal, which has an effect on the IS–LM diagram because money demand (and hence the LM curve) depends on the nominal interest rate, while investment (and hence the IS curve) depends on the real interest rate. Suppose the IS–LM diagram is drawn with the nominal interest rate on the vertical axis. In this case, the LM curve is unaffected by the change in π^e. However, for each nominal interest rate i, the real interest $r = i - \pi^e$ implied by the Fisher equation is increased because $\pi^e < 0$. A higher real interest rate discourages investment, which means a reduction in demand that shifts the IS curve to the left. The precise size of the shift can be determined by noting that if i fell by as much as π^e, the real interest rate and the level of demand would remain the same. Therefore, the IS curve shifts down by an amount equal to the decrease in π^e.



In equilibrium, output falls and the nominal interest rate declines. It can be seen from the diagram that the nominal interest rate declines by less than inflation expectations, so the real interest rate rises, as is consistent with the fall in output.

Now suppose monetary policy satisfies the Taylor principle. This means that expansionary monetary policy must reduce the nominal interest rate by more than the fall in inflation expectations. In the IS–LM model, this can be achieved by an increase in the money supply, shifting the LM curve to the right. In this case, since the fall in the nominal interest rate exceeds the fall in inflation expectations, the Fisher equation implies the real interest rate falls. To achieve this, the shift of the LM curve to the right must dominate the shift of the IS curve to the left, so output rises.



Question 11

Consider the sticky-wage model of aggregate supply. Nominal wages W are contractually fixed at $W = \bar{W}$. Firms are perfectly competitive and the price level P is flexible (both firms and workers are fully informed about the level of prices P).

Output Y is produced according to the production function $Y = F(L)$, where L is employment. Employment is chosen by firms to maximise profits. The marginal product of labour is diminishing. Workers desired labour supply is constant at L^*.

(a) [6 marks]

i. Explain why firms' demand for labour is determined by the condition $w = F'(L)$, where $w = W/P$ is the real wage and $F'(L)$ is the marginal product of labour. Show in a diagram how employment L and unemployment $L^* - L$ are found given a price level P.

ii. If wages were flexible, how would equilibrium employment and unemployment be different? Would the flexible-wages level of employment depend on the price level P? Explain.

(b) [7 marks] Show how the short-run aggregate supply (SRAS) curve is derived when nominal wages are sticky. Using the AD–AS model, find the effects of an increase in the money supply on employment, unemployment, output, the price level, real wages, and real profits.

(c) [7 marks] Assume that the production function implies the marginal product of labour is proportional to the average product of labour, that is, $F'(L) = \gamma Y/L$, where γ is a positive constant. Suppose the economy is subject to both demand shocks (shifts of the IS, and hence AD, curves) and supply shocks (changes in productivity Y/L). Find a monetary policy target that stabilises unemployment even though nominal wages are sticky. *[Hint: Find what condition must be satisfied if monetary policy achieves its objective.]*

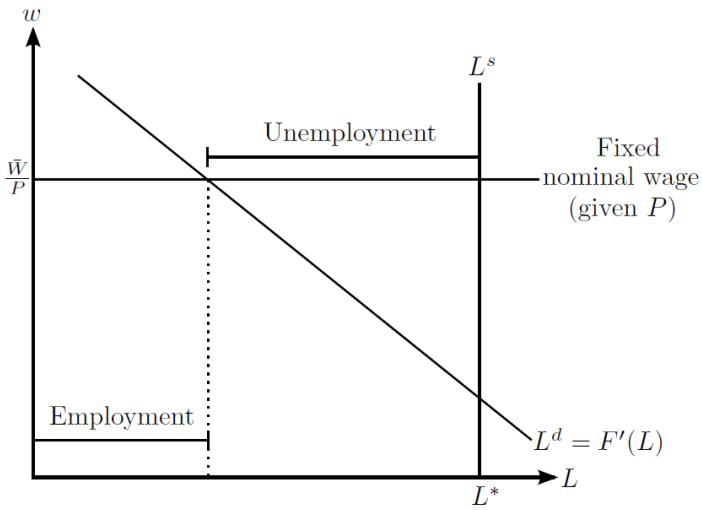
Reading for this question

Subject guide, Chapters 3 and 12.

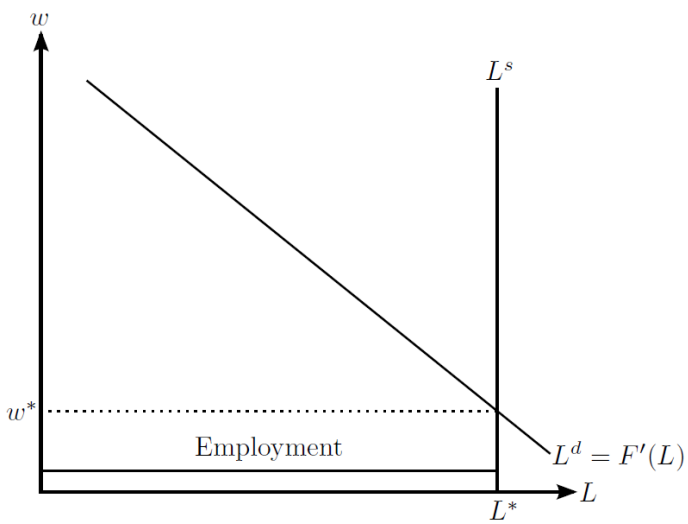
Approaching the question

(a) A perfectly competitive firm can sell as much as it likes at the prevailing price P. Hiring an extra unit of labour yields an extra $F'(L)$ units of output (the marginal product of labour), which sell for $PF'(L)$. The extra unit of labour costs W. The firm increases profits by hiring more labour if $PF'(L) > W$, or equivalently, $F'(L) > w$, where $w = W/P$ is the real wage. It is rational for the firm to hire workers up to the point where profits cannot be increased any further, namely, $F'(L) = w$, where the marginal product of labour has declined to the point where it is equal to the real wage. The marginal product of labour schedule thus defines the firm's demand curve for labour. The desired labour supply of workers is inelastic at L^*.

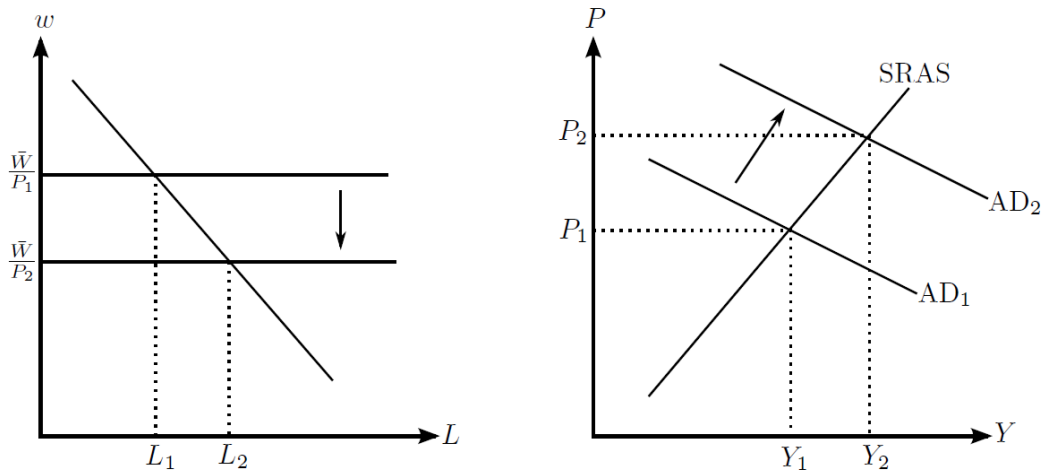
If nominal wages are fixed at $W = \bar{W}$, the real wage w is determined by the price level P. At the prevailing real wage, if labour demand falls short of labour supply L^* then employment is the profit-maximising level of labour demand L. The difference between L^* and L is unemployment. Note that the case where desired labour demand exceeds labour supply is ignored here.



If wages are flexible then the nominal wage adjusts so that the implied real wage equates the demand for labour and the supply of labour. Since labour supply is constant and labour demand depends only on the real wage, the price level has no effect on equilibrium unemployment (only real variables matter: there is no money illusion).



(b) An increase in the price level lowers the real wage if nominal wages remain sticky. This raises the demand for labour, and thus employment. Unemployment decreases because desired labour supply is unchanged but actual employment rises. With more employment, more output is produced. This means there is a positive relationship between prices and output: the short-run aggregate supply curve.



An increase in the money supply implies a rightward shift of the aggregate demand curve. This leads to a movement along the short-run aggregate supply curve, increasing the price level and output. In the labour-market diagram, the real wage falls (as the nominal wage is sticky). This leads to an increase in employment consistent with the rise in output. Higher employment implies a fall in unemployment. Since real wages fall and output and employment rise, the real profits of firms must increase.

(c) If the marginal product of labour is $F'(L) = \gamma Y/L$ then the profit-maximising level of employment is determined by:

$$w = \gamma \frac{Y}{L}.$$

Given the sticky nominal wage $W = \bar{W}$, the real wage is $w = \bar{W}/P$, hence:

$$\frac{\bar{W}}{P} = \gamma \frac{Y}{L}.$$

Success in stabilising unemployment is ensuring that $U = L^* - L$ is constant at some level U_0, which requires:

$$L = L^* - U_0.$$

Substituting this into the labour demand equation yields:

$$\frac{\bar{W}}{P} = \gamma \frac{Y}{L^* - U_0}.$$

Note that this equation can be rearranged as follows:

$$PY = \frac{\bar{W}(L^* - U_0)}{\gamma}.$$

The left-hand side is nominal GDP, and the right-hand side must be constant given fixed nominal wages and the desire for a stable level of unemployment. The equation therefore shows that this goal requires a monetary policy that stabilises nominal GDP.

Question 12

Consider the following consumption choice problem. A household has no initial assets, receives income Y_1, and pays (lump-sum) tax T_1. The household chooses its current consumption C_1 and the amount of wealth W to bequeath to its children. The children receive bequests of value $(1 + r)W$ and will receive income Y_2 and pay

taxes T_2 in the future, where r is the real interest rate. Ignoring grandchildren and subsequent generations, the children's consumption C_2 will be:

$$C_2 = Y_2 - T_2 + (1 + r)W.$$

The government has current expenditure G_1 and plans expenditure G_2 in the future (when today's children have become taxpayers). If the government runs a deficit $D = G_1 - T_1$ then it issues bonds that pay interest rate r. Assume that the government has no initial debt and must repay all debt by the next generation, that is:

$$T_2 = G_2 + (1 + r)D.$$

(a) [6 marks] Write down a budget constraint including C_1 and W for the current-generation household and derive a present-value budget constraint for the household's current consumption C_1 and the consumption C_2 of its children. Write down the current budget constraint for the government and derive the government's present-value ('life-time') budget constraint.

(b) [7 marks] Suppose the household is altruistic and obtains utility from both its current consumption C_1 and the anticipated consumption C_2 of its children.

i. Draw indifference curves over C_1 and C_2 and the present-value budget constraint and explain how the household optimally determines consumption C_1 and bequests W *[Hint: the consumption choice is analogous to the two-period Fisher model]*.

ii. Find what happens to C_1, C_2, and W if the government increases spending G_1 without raising taxes T_1.

(c) [7 marks] Now suppose the household is selfish and obtains utility only from its current consumption C_1 and not from its children's consumption C_2. Assume it is not possible to leave negative bequests (i.e. to bequeath debt, where $W < 0$).

i. How are the household's indifference curves and present-value budget constraint different compared to part (b)?

ii. What are the effects on C_1, C_2, and W of an increase in government spending G_1 with taxes T_1 left unchanged? Explain your answer.

Reading for this question

Subject guide, Chapters 9 and 12.

Approaching the question

(a) The budget constraint of the current generation is:

$$C_1 + W = Y_1 - T_1$$

which states that the sum of the current generation's consumption and its bequests must sum to its income after taxes. The consumption of the next generation will be:

$$C_2 = Y_2 - T_2 + (1 + r)W$$

that is, the sum of the next generation's income and the value (with interest) of the bequests it receives. The current-generation budget constraint implies $W = Y_1 - T_1 - C_1$, which can be substituted into the next generation's budget constraint:

$$C_2 = Y_2 - T_2 + (1 + r)(Y_1 - T_1 - C_1).$$

Dividing both sides by $1 + r$ and rearranging leads to an intergenerational budget constraint:

$$C_1 + \frac{C_2}{1 + r} = (Y_1 - T_1) + \frac{Y_2 - T_2}{1 + r}.$$

The government's budget constraint on taxes and spending for the current generation requires that expenditure must be financed either by taxes or borrowing:

$$G_1 = T_1 + D.$$

To cover future expenditure and repay debt, future taxes must be:

$$T_2 = G_2 + (1 + r)D.$$

Substituting $D = G_1 - T_1$ from the current budget constraint leads to:

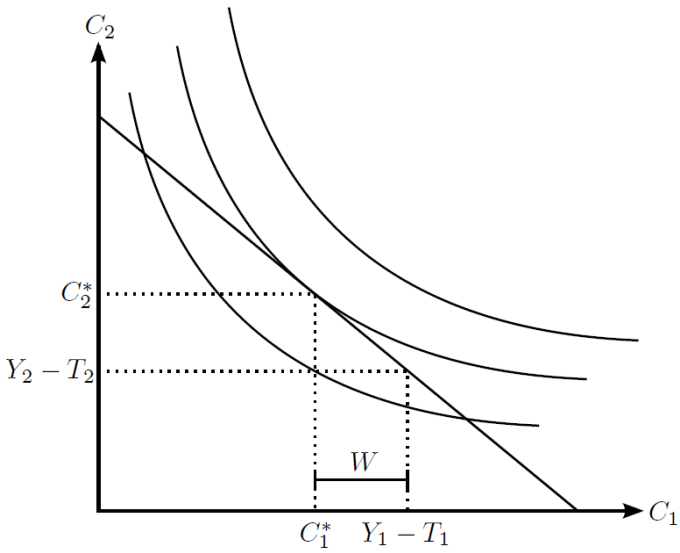
$$T_2 = G_2 + (1 + r)(G_1 - T_1)$$

and then dividing both sides by $1 + r$ and rearranging:

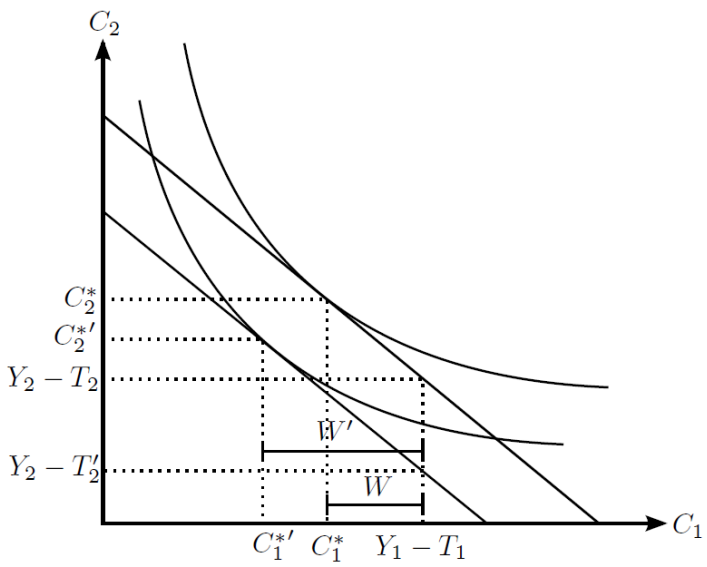
$$G_1 + \frac{G_2}{1 + r} = T_1 + \frac{T_2}{1 + r}.$$

This is the government's life-time budget constraint.

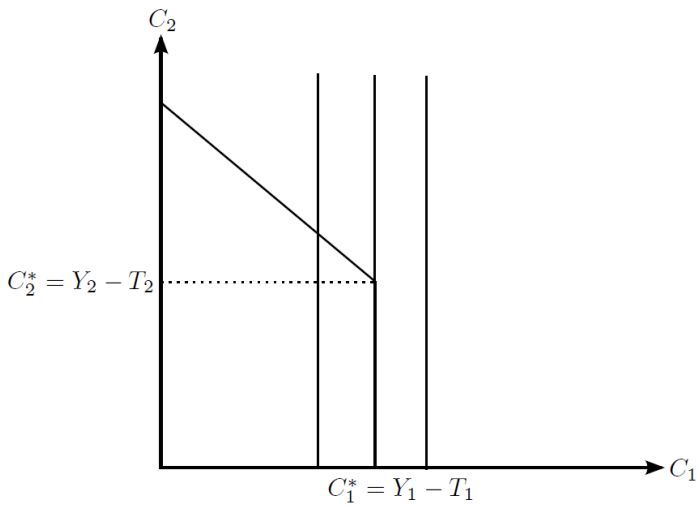
(b) Since the current generation is altruistic, its utility depends on the next generation's consumption C_2 as well as its own consumption C_1. Indifference curves representing combinations of C_1 and C_2 that yield the same utility can then be drawn in the usual way. Through its choice of C_1, the current generation implicitly determines its bequests W. It has been shown that C_1 and C_2 must satisfy a present discounted value budget constraint given current and future incomes after tax. This corresponds to a downward sloping straight line with gradient $-(1 + r)$. Maximum utility for the current generation (taking account of its altruistic concern for the next generation) is attained at the point where the indifference curve is tangent to the budget line.



If the government were to raise spending G_1 without increasing taxes T_1 then there is an increase in the budget deficit which means higher taxes in the future are required to repay government debt. This implies a shift of the budget constraint to the left: for each level of C_1, the same bequests W result in lower consumption for the future generation because T_2 rises. The current generation that cares about its own consumption and the future generation's consumption will spread the fall in consumption between C_1 and C_2 (technically, all generations' consumptions are assumed to be normal goods). To avoid the future generation facing the full burden of the higher taxes, the current generation increases its bequests, though not by enough to avoid some fall in C_2.

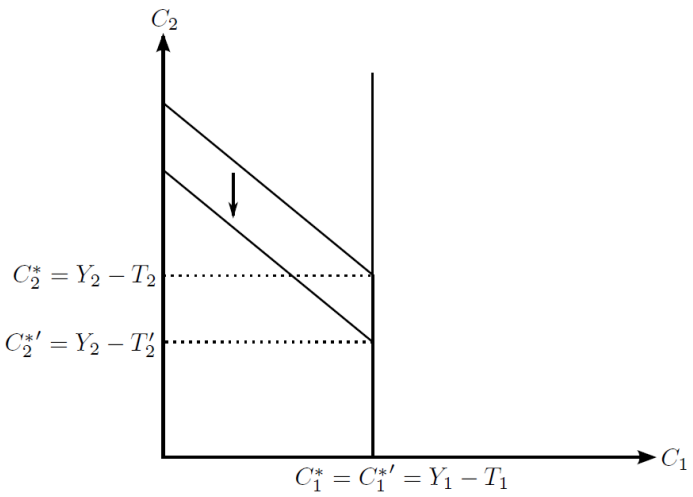


(c) There are two changes to the consumption choice diagram from part (b). First, because the current generation is selfish, its utility depends only on C_1. This means that the indifference 'curves' are vertical lines (given its own consumption C_1, the current generation is indifferent between all possible values of consumption C_2 of the next generation). The second change is due to the impossibility of leaving negative bequests (bequeathing debt). Since bequests are $W = Y_1 - T_1 - C_1$, the requirement that $W \geq 0$ means the current generation's consumption C_1 cannot exceed its disposable income $Y_1 - T_1$. This truncates the usual budget constraint to the right of current disposable income.



Given the shape of the indifference curves, utility maximisation for the current generation means choosing a point as far as possible to the right. This is always where bequests are zero (at the point where the budget constraint is truncated).

If there is an increase in current government spending then this causes the budget line to shift to the left (taxes must rise to pay for this). In the question, there is no change in current taxes, so T_2 rises. With no change in current disposable income, the current consumption level at which the budget line is truncated is unchanged. This means the selfish current generation does not change either its consumption or bequests (it continues to leave zero bequests, consuming all its disposable income). The future generation faces a higher tax burden and must reduce its consumption C_2.



Question 13

Consider the Solow growth model. Output Y is produced according to the production function $Y = F(K, L)$, where K is the capital stock and L is the labour force. The function $F(K, L)$ has constant returns to scale and diminishing marginal returns to capital. The labour force and technology are both constant over time ($n = 0$ and $g = 0$, and the level A of technology is set to 1 in the production function). Investment I is equal to saving, which is a fraction s of income. Capital depreciates at rate δ. The evolution of the capital stock over time is determined by the equation $\Delta K = I - \delta K$.

(a) [6 marks] Let $y = Y/L$ and $k = K/L$ denote output per worker and capital per worker.

i. Show that $y = f(k)$, where $f(k)$ is the 'per worker' production function, and sketch the function $f(k)$, explaining its shape. Now derive an equation for the change over time in capital per worker Δk in terms of $f(k)$, the saving rate s, and the depreciation rate δ.

ii. Using a diagram, explain why there is a steady state for capital per worker and output per worker.

(b) [7 marks] Assume all countries in the world have the same production function $f(k)$, the same depreciation rate δ, and the same saving rate s. Consider two countries, a richer country that has reached its steady state, and a poorer country that begins below its steady state. For the two countries, plot graphs of the following variables over time: output per worker, capital per worker, and the capital-output ratio. Use your analysis to answer the following questions:

i. Does growth occur in the two countries? How long does any growth last for?

ii. Is there convergence (in income levels and/or growth rates) between the poor and rich countries?

Would any of your answers be different if the rich country increased its saving rate above the saving rate in the poor country?

(c) [7 marks] Now consider the 'AK' model of economic growth. The production function is now $Y = K$, but all other assumptions of the Solow model are unchanged (and it is assumed that $s > \delta$). Assume all countries in the world have the same production function and the same values of δ and s. Considering again a poor and a rich country, use the 'AK' model to answer questions (i) and (ii) posed in part (b).

Reading for this question

Subject guide, Chapters 5 and 6.

Approaching the question

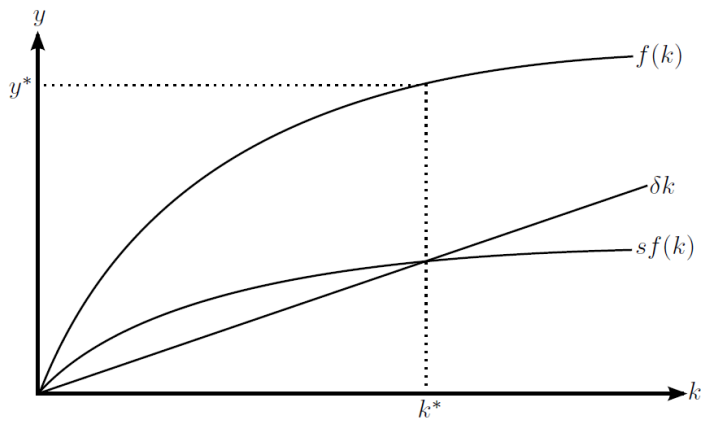
(a) The level of output per worker is:

$$y = \frac{Y}{L} = \frac{F(K,L)}{L} = F\left(\frac{K}{L}, \frac{L}{L}\right) = F(k,1) = f(k)$$

where $f(k)$ is the per-worker production function, which gives y as a function of the capital-labour ratio k. The first derivative of this function is

$$f'(k) = \frac{\partial F(k,1)}{\partial k}$$

which is the marginal product of capital. Since the marginal product of capital is positive but diminishing, it follows that the per-worker production function $f(k)$ is increasing, but at a diminishing rate. This means it has a concave shape, as shown in the diagram below.



The labour force L is constant over time, so the change in the capital-labour ratio is $\Delta k = \Delta K/L$. Substituting the equation for the change in the capital stock ΔK and using $I = sY$ and $y = f(k)$:

$$\Delta k = \frac{I - \delta K}{L} = \frac{sY - \delta K}{L} = sy - \delta k = sf(k) - \delta k.$$

A steady state of the model is where the capital-labour ratio is constant over time, that is, where $\Delta k = 0$. Since $\Delta k = sf(k) - \delta k$, this requires:

$$sf(k) = \delta k.$$

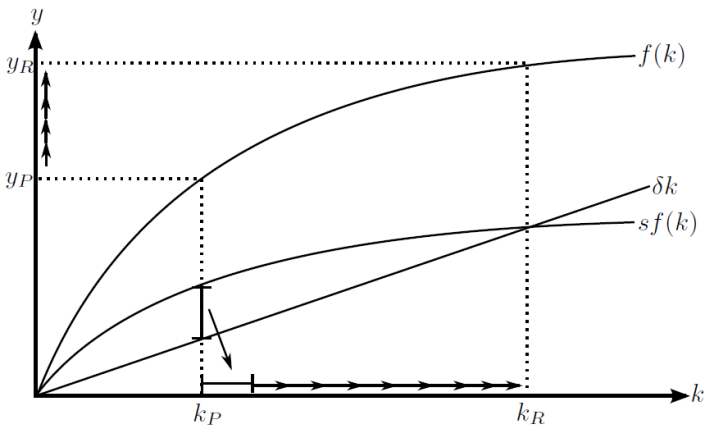
Geometrically, the steady state is found where the line $sf(k)$ intersects the line δk. The former is a scaled down version of the production function, having the same shape (increasing, but at a diminishing rate). The latter is a straight line with gradient δ. With the gradient of $sf(k)$ falling as k increases, this line eventually intersects the line δk (strictly speaking, as long as its gradient would eventually fall to zero). This yields the steady-state capital-labour ratio k^*.

Since $y = f(k)$, output per worker y is constant over time if k is constant. Therefore, in the steady state $k = k^*$, output per worker is constant at y^*.

(b) Let k_R denote the capital-labour ratio of the rich economy, which is equal to the steady-state k^*, and is associated with output per worker $y_R = y^*$. The poor country has income $y_P < y_R$, and since both economies have the same per-worker production function $y = f(k)$, the poor country must have a lower capital-labour ratio $k_P < k_R = k^*$ than the rich country.

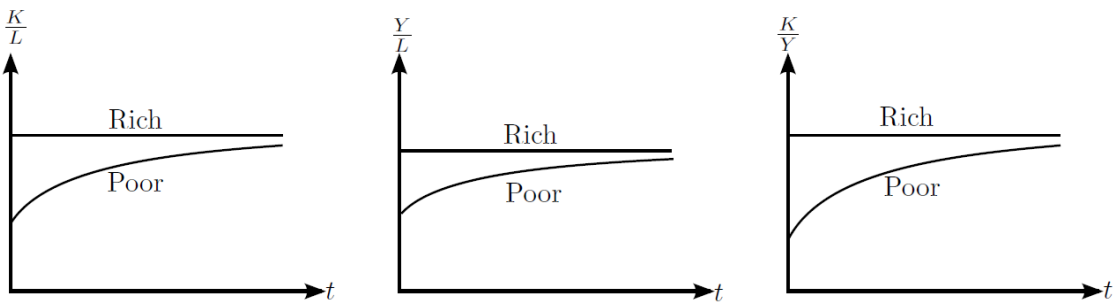
Since the two countries share the same production function $f(k)$ and the same saving and depreciation rates s and δ, they both have the same steady-state values of k and y.

The rich country has already reached its steady state, so there are no further changes in its capital-labour ratio or output per worker. The ratio of capital to output is K/Y, which is the same as the ratio k/y. With k and y constant over time, the capital-output ratio is also constant.



The poor country begins with a capital-labour ratio below its steady state. Since the saving line $sf(k)$ is above the depreciation line δk at this point, and as $\delta k = sf(k) - \delta k$, the poor country has savings in excess of the amount needed to cover depreciation, which allows the capital-labour ratio to increase. This process continues until the poor country reaches its steady state. As the capital-labour ratio rises, the economy moves along the production function, leading to increases in output per worker. Given the shape of the production function, an increase in k lowers the ratio y/k (the gradient of a ray from the origin to the point on the production function), and thus increases the ratio k/y. Since this ratio is also equal to K/Y, the poor country experiences a rise in its capital-output ratio over time.

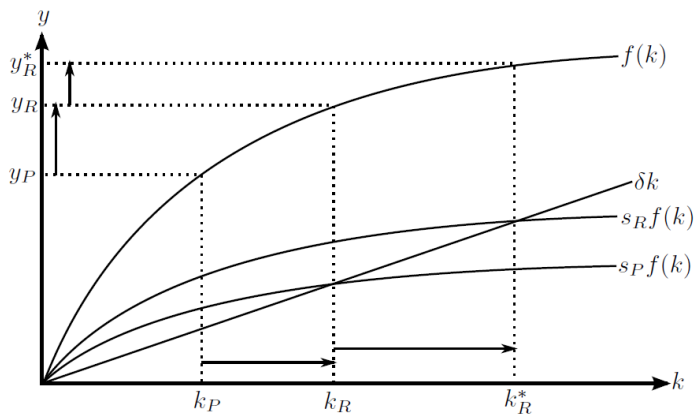
The time paths of K/L, Y/L, and K/Y for the two countries are depicted below.



The rich economy experiences no growth in incomes because it has already reached its steady state. At the steady state, there is no scope to sustain a higher capital stock, and thus incomes stagnate in the absence of any source of growth other than capital accumulation. The poor country experiences growth in output per worker during the process of convergence to its steady state. During this time, the capital-labour ratio is increased, which makes workers more productive. However, once the steady state is reached, growth comes to a halt. In this example, there is full convergence both in growth rates and levels of output per worker between the poor and rich countries. In the long run, both economies share zero growth and the same level of living standards.

If the rich country were to increase its saving rate to s_R (the poor country's saving rate

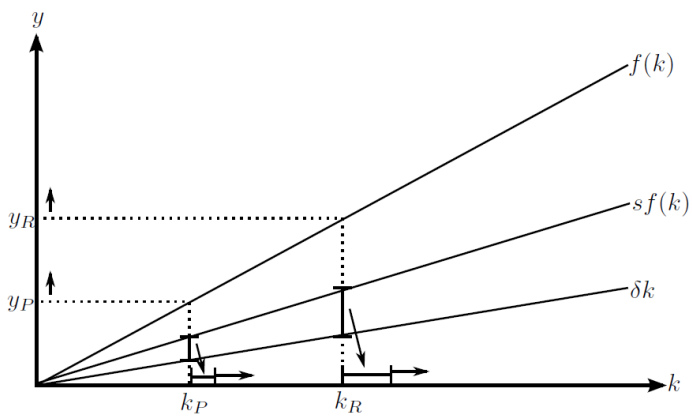
remains at s_P) then the rich country would have a higher steady-state capital-labour ratio (the saving line shifts up, and now intersects the depreciation line at a higher level of k). It would also have a higher steady-state level of output per worker because $y = f(k)$. By increasing its saving rate, the richer country is able to enjoy a spell of growth in output per worker while it converges to its new steady state. Both poor and rich countries now experience growth during a transitional period, but growth still stops once they reach their steady states. Since growth rates ultimately fall to zero in both countries, there is still convergence between them in terms of growth rates. However, levels of output per worker do not converge because the richer country has a higher level of output per worker in its steady state.



(c) The AK production function (with $A = 1$) implies $f(k) = K/L = k$. This is linear in k. The implications of the AK model can be analysed using the same tools as the Solow model, but with the concave $f(k)$ replaced by a straight line. Since $f(k)$ is linear, the saving line $sf(k) = sk$ is also now a straight line. Since $\Delta k = sk - \delta k = (s - \delta)k$ and $s > \delta$, there is no value of k where the saving line sk intersects the depreciation line δk (except at the uninteresting case of $k = 0$). This means there is no steady-state capital-labour ratio, and since $y = k$, no steady-state level of output per worker. Growth is determined by:

$$\frac{\Delta y}{y} = \frac{\Delta k}{k} = \frac{sk - \delta k}{k} = s - \delta.$$

The growth process is depicted in the diagram below.



Since $s > \delta$, growth is positive and continues perpetually. This is true in both the poor and rich countries. Note that both countries have the same growth rate because they share the same values of s and δ. While there is convergence between countries in growth rates, the

poor country never attains the same level of output per worker as the rich country, so there is no convergence in levels of income.

Question 14

Consider the neoclassical model of investment. There is a production function for output $Y = F(K)$ that depends on the stock of capital K. The capital stock is

$$K = (1 - \delta)K_0 + I$$

where I is investment spending, K_0 is the past capital stock, and δ is the rate of depreciation of capital. The optimal level of the capital stock is determined by the equation:

$$F'(K) = r + \delta$$

where $F'(K)$ is the marginal product of capital and r is the real interest rate.

Assume that the production function is $Y = A\sqrt{K}\sqrt{L}$, where A is the level of (exogenous) total factor productivity, and L is the size of the labour force (also exogenous).

(a) [6 marks]

i. Derive an expression for the marginal product of capital for the production function given above and show that it is diminishing as the capital stock increases.

ii. Show how the optimal capital stock K can be determined using a diagram, and explain how this determines the level of investment spending in the short run (when the past capital stock K_0 is fixed) and in the long run (when $K = K_0$).

(b) [7 marks] Explain how the level of investment in the short run and the long run is affected by the following:

i. Discovery of new technologies raising total factor productivity A.

ii. A war that destroys a portion of the existing capital stock K_0.

iii. Adverse climate change that increases the cost of maintaining buildings (interpreted as an increase in the rate of depreciation δ).

(c) [7 marks] Suppose output Y is used either for consumption C or investment I, so $C + I = Y$. Explain how to find the capital stock that maximises consumption in the long run (when $K = K_0$). Find the real interest rate that would provide incentives for firms to undertake the level of investment required to reach this capital stock.

Reading for this question

Subject guide, Chapters 5 and 10.

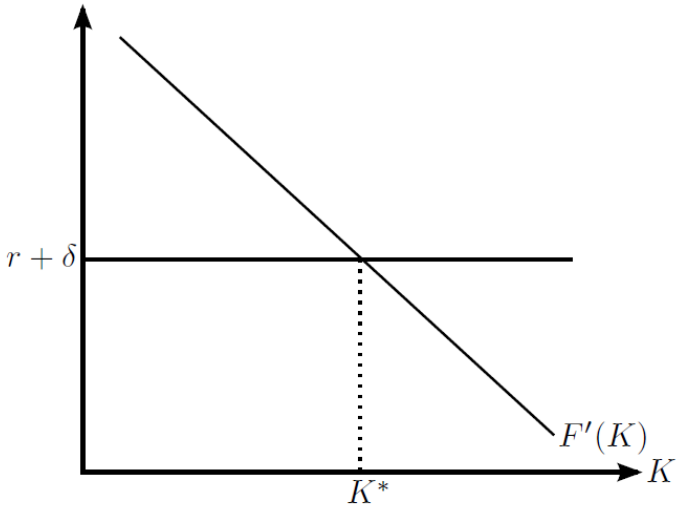
Approaching the question

(a) The marginal product of capital for the production function given in the question is:

$$F'(K) = \frac{\partial Y}{\partial K} = \frac{A\sqrt{L}}{2\sqrt{K}}.$$

This is a decreasing function of the capital stock, so the marginal product of capital is diminishing.

The optimal capital stock can be found using a diagram where the marginal product of capital $F'(K)$ is plotted against the capital stock K. The optimal level of the capital stock K^* is where the height of the marginal product schedule is equal to $r + \delta$.



Once K^* is determined, investment in the short run (starting from a capital stock K_0 determined in the past) is:

$$I = K^* - (1 - \delta)K_0$$

that is, investment is the amount needed for the capital stock to reach K^* given the starting point $(1 - \delta)K_0$. In the long run the optimal capital stock will have been reached, so $K = K_0$, and the optimal level of investment is what is required to maintain the capital stock at K^*:

$$I = K^* - (1 - \delta)K^* = \delta K^*.$$

(b) An increase in total factor productivity owing to new technologies (a rise in A) shifts the marginal product of capital curve to the right (the marginal product of capital is increasing in total factor productivity A). This increases the optimal capital stock and raises investment in both the short run and the long run.

The destruction of existing capital in a war reduces the initial capital stock K_0. This itself has no effect on the marginal product of capital curve or on r or δ. It follows that the optimal capital stock is unchanged. However, the initial level of capital is now too low, so investment rises in the short run to return the economy to K^*. But once this point is reached, investment returns to normal levels, so there is no long-run effect on investment.

A rise in the depreciation rate of capital owing to adverse climate change shifts the $r + \delta$ line upwards, which reduces the optimal capital stock K^*. This tends to reduce investment in both the short run and the long run, but the increase in δ has an offsetting effect because more investment is needed to maintain a given capital stock. It is unclear whether investment rises or falls in either the short run or the long run.

(c) Since $C + I = Y$, consumption is equal to:

$$C = Y - I.$$

In the long run, $K = K_0$, so investment is equal to $I = \delta K$. Output is given by $Y = F(K)$. Therefore, long-run consumption as a function of the capital stock K is:

$$C = F(K) - \delta K.$$

Taking the derivative with respect to K and setting this to zero gives the first-order condition for maximising long-run consumption:

$$F'(K) - \delta = 0$$

and hence $F'(K) = \delta$. The consumption-maximising capital stock has a marginal product $F'(K)$ equal to the depreciation rate δ. Since firms choose investment such that $F'(K) = r + \delta$, achieving this level of the capital stock requires a real interest rate of zero.